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Jump Discontinuity Equations in Cake Filtration

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ABSTRACT

Jump discontinuity balances in continuum theory are well known and are frequently applied in the literature for single-phase systems. Jump discontinuity balances between dispersed multiphase regions have been derived for multiphase volume averaging continuum theory. However, multiphase region jump balances have not received comparable attention in the literature in multiphase applications. In this work the continuum equations and jump balances are summarized and compared for the cake filtration and cake drainage processes under air pressure. The comparison shows the jump conditions for the fluid and solid phases are easily decoupled in cake filtration. On the other hand, during the drainage process the mass discontinuity balances for the gas and liquid phases are coupled if there is significant mass transfer between the phases at the drainage boundary. Furthermore, the momentum discontinuity balances are also coupled when the capillary forces are significant.

INTRODUCTION

The derivation and interpretation of the volume-averaged continuum equations are well established (1–4). Continuum models for cake filtration are frequently reported in literature [e.g., Tiller (5), Wakeman (6), and Chase and Willis (7)]. The equations for cake filtration are evaluated using local experimental data (pressure and porosity) which make it possible to apply the continuum equations with only concern for the jump conditions between the cake and the slurry approaching the cake (7, 8).

Continuum models for evaluating the pressure loss across the whole filter assembly must include the resistance to flow through the filter medium and any support structures such as the wire mesh that may be used to provide structural support to paper and cloth media. The two-resistance model traditionally applied in the literature (9), in which the cake and the medium resistances are summed to give the overall resistance, accounts for the medium resistance provided that the effect of the cake on the medium is included in the evaluation (10).

The addition of the two resistances in the traditional model has inherent assumptions about the jump discontinuity conditions between the multiphase regions. While some of the conditions may appear trivial for cake filtration when only one fluid phase is present, the purpose of this paper from an academic perspective is to bring attention to these jump conditions and how they can differ when air displaces the liquid phase during drainage.

The cake filtration process considered here is the simple rectilinear one-dimensional pressure filter shown in Fig. 1. In this process there are five distinct regions which have significantly different material properties

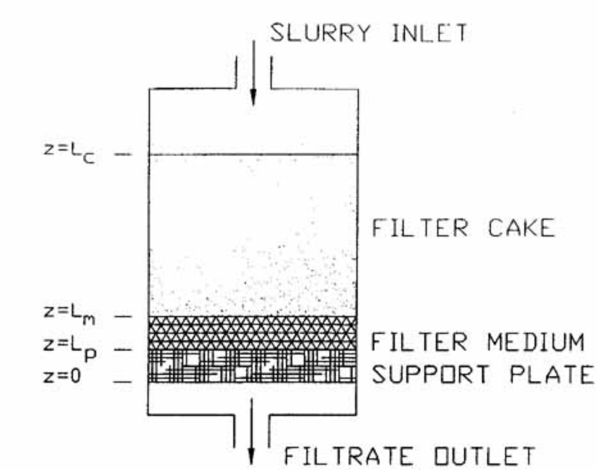

FIG. 1 One-dimensional rectilinear flow filter cake assembly. Five distinct regions are the slurry above the cake, the cake, the filter medium, the support plate, and the filtrate. The boundaries between the regions are identified by their position in the z-direction as indicated on the left side of the figure.

(such as differences in materials, porosity, and pore size). The different regions are the slurry above the cake, the cake, the filter medium, the support plate under the medium, and the filtrate. The discontinuities or boundaries between the regions are identified at the marked positions in the z direction in Fig. 1: as the boundary between the slurry and cake at $z = L_c$, as the boundary between the cake and medium at $z = L_m$, as the boundary between the medium and the support plate at $z = L_p$, and as the boundary between the support plate and the filtrate at $z = 0$.

During the drainage process a gas such as air displaces the liquid phase in the cake, as shown in Fig. 2. Shown in Fig. 2 within the cake is the drainage boundary between the portion of the cake filled with the gas (with residual liquid trapped in pores or adsorbed onto the solid surface) and the portion of the cake saturated with the liquid phase. This drainage boundary moves downward with velocity w_z. This boundary identifies one more discontinuity that must be included in the drainage model. This drainage boundary is modeled here as being within the cake only. This boundary condition can easily be extended to the filter medium and support plate if necessary.

These jump balances will be combined in future papers with the continuum balances for cake filtration and other similar multiregioned-dispersed-multiphase processes. These balances will help us to account for the interactions between the regions, such as when clogging occurs on a filter media, and give us more tools for predicting and interpreting experimental data.

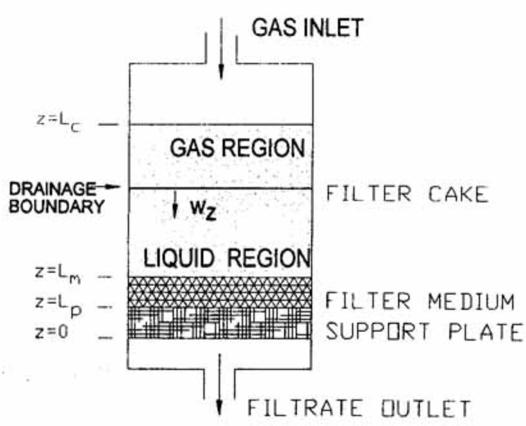

FIG. 2 Jump discontinuity within the filter cake due to the displacement of the liquid phase by a gas.

PHASE AND REGION CONTINUUM EQUATIONS

The α-phase mass (Eq. 1) and momentum (Eq. 2) balances from continuum theory are

$$\frac{\partial(\epsilon^{\alpha}\rho^{\alpha})}{\partial t} + \nabla\cdot(\epsilon^{\alpha}\rho^{\alpha}\mathbf{v}^{\alpha}) = 0 \tag{1}$$

$$\frac{\partial(\epsilon^{\alpha}\rho^{\alpha}\mathbf{v}^{\alpha})}{\partial t} + \nabla\cdot(\epsilon^{\alpha}\rho^{\alpha}\mathbf{v}^{\alpha}\mathbf{v}^{\alpha}) + \epsilon^{\alpha}\nabla P^{\mathrm{f}} + \mathbf{F}^{\alpha} + \nabla\boldsymbol{\tau}^{\alpha} - \epsilon^{\alpha}\rho^{\alpha}\mathbf{g} = 0 \tag{2}$$

for a two-phase system with no mass transfer and no chemical reactions (3). The mass balance in Eq. (1) has terms accounting for the accumulation and convection of mass. The momentum balance in Eq. (2) has terms, from left to right, accounting for the accumulation of inertia, inertial convection, pressure force, drag force between the phases, the deviatoric stress term, and the force of gravity.

Equations (1) and (2) are the basis for the equations derived here. The process is assumed to be a one-dimensional flow system with variations in the z-direction and uniformity in all other directions. Also, the intrinsic phase densities, ρ^{α}, are assumed to be constant. The mass balance of Eq. 1 reduces to

$$\frac{\partial\epsilon^{\alpha}}{\partial t} + \frac{\partial(\epsilon^{\alpha}v_{\mathrm{z}}^{\alpha})}{\partial \mathrm{z}} = 0 \tag{3}$$

The momentum balance in Eq. (2) also simplifies. Willis et al. (11) deduced through dimensional analysis that the inertial terms and the fluid stress term are insignificant compared to the pressure and drag force terms. Neglecting the insignificant terms, then for the one-dimensional flow problem considered here, the momentum balance in Eq. (2), reduces to

$$\epsilon^{\mathrm{f}}\frac{\partial P}{\partial z} + F_{\mathrm{z}}^{\mathrm{d}} = 0 \tag{4}$$

for the fluid-phase momentum, and to

$$\epsilon^{\mathrm{s}}\frac{\partial P}{\partial z} + \frac{\partial\tau_{\mathrm{zz}}^{\mathrm{s}}}{\partial z} - \epsilon^{\mathrm{s}}(\rho^{\mathrm{s}} - \rho^{\mathrm{f}})g_{\mathrm{z}} - F_{\mathrm{z}}^{\mathrm{d}} = 0 \tag{5}$$

for the solid-phase momentum, where the piezometric pressure, P, is the combined fluid phase pore pressure and the gravity force on the fluid phase

$$P = P^{\mathrm{f}} - \rho^{\mathrm{f}}g_{\mathrm{z}}z \tag{6}$$

The phase mass balances in Eq. (3) and the phase momentum balances in Eqs. (4) and (5) apply to each of the regions (cake, filter medium, and porous plate). Though the equations are the same, the regions may behave differently due to their material makeup. The material properties which make the different regions distinguishable are introduced through constitutive relations for the interphase drag force, F_z^d, and the solid phase stress, τ_{zz}^s.

The phase balances are summed to obtain the region balances. The region mass balance is

$$\frac{\partial}{\partial z}(\epsilon^f v_z^f + \epsilon^s v_z^s) = 0 \tag{7}$$

where the sum of the volume fractions is unity, $\epsilon^f + \epsilon^s = 1$. This region mass balance indicates that the quantity $(\epsilon^f v_z^f + \epsilon^s v_z^s)$ is independent of the z position though it may still be a function of time.

The region momentum balance is

$$\frac{\partial P}{\partial z} + \frac{\partial \tau_{zz}^s}{\partial z} - \epsilon^s(\rho^s - \rho^f)g_z = 0 \tag{8}$$

which balances the pressure force, the solid stress, and the gravitational (buoyant) forces.

The above phase and region continuum balances account for the kinematics and dynamics of the materials within a given region. They are applicable to the cake filtration process shown in Fig. 1. For the drainage process, Eqs. (1) and (3) must be modified to include a term for the mass transfer of residual liquid from the solid matrix to the gas phase if the mass transfer is significant. Also, the residual liquid effects on the drag force between the phases must be accounted for such as through the relative permeability (12).

The transfer of mass and momentum across a boundary between two regions is accounted for in the region jump discontinuity balances which are now considered.

DISCONTINUITY BALANCES BETWEEN REGIONS

The continuum scale equation for an arbitrary property ϕ is given as

$$\frac{\partial(\rho\phi)}{\partial t} + \nabla\cdot(\rho\phi\mathbf{v}) + \nabla\cdot\mathbf{i} - \rho(f + g) = 0 \tag{9}$$

where $\mathbf{i}$ is the flux of property ϕ and the quantities f and g represent the body and external sources of property ϕ. The jump balance is obtained

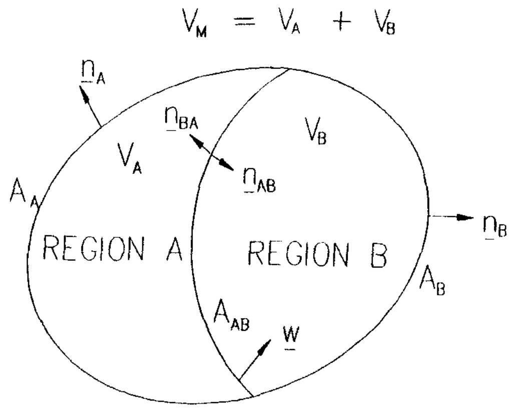

FIG. 3 Material volume containing two regions separated by surface A_{AB}.

by integrating Eq. (9) three times, once over each of the region volumes V_A and V_B, and once over the combined material volume, V_m, shown in Fig. 3. When the two integral equations obtained from the region volumes are subtracted from the integral equation for the combined material volume, the general jump balance is obtained (4):

$$[\rho_A\phi_A(\mathbf{w} - \mathbf{v}_A) - \mathbf{i}_A]\cdot\mathbf{n}_{AB} - [\rho_B\phi_B(\mathbf{w} - \mathbf{v}_B) - \mathbf{i}_B]\cdot\mathbf{n}_{AB} = 0 \quad (10)$$

where subscripts A and B refer to the two regions, $\mathbf{w}$ is the velocity of the moving boundary (surface A_{AB}), and $\mathbf{n}_{AB}$ is the unit area normal vector for the boundary between the two regions.

For most processes there is not a generation of property ϕ at the boundary, and the right-hand side of Eq. (10) is zero. However, for processes in which a generation may occur, such as a chemical reaction at the boundary, then the zero on the right-hand side of the equation would be replaced with a generation term.

MASS DISCONTINUITY BALANCES FOR CAKE FILTRATION

The region mass discontinuity balance is

$$[\rho_A(\mathbf{v}_A - \mathbf{w}) - \rho_B(\mathbf{v}_B - \mathbf{w})]\cdot\mathbf{n}_{AB} = 0 \quad (11)$$

When only two phases (fluid and solid) are present, the region densities and velocities are related to the phase intrinsic densities and volume mass averaged velocities by

$$\rho = \epsilon^f \rho^f + \epsilon^s \rho^s \tag{12}$$

$$\rho \mathbf{v} = \epsilon^f \rho^f \mathbf{v}^f + \epsilon^s \rho^s \mathbf{v}^s \tag{13}$$

When there is no mass transfer between the phases, the convections of each phase across the boundary are linearly independent of each other. Equations (12) and (13) can be substituted into Eq. (10) and the linear independence applied to separate the terms for each of the phases to obtain

$$[\epsilon_A^f (v_{Az}^f - w_z) - \epsilon_B^f (v_{Bz}^f - w_z)] = 0 \tag{14}$$

$$[\epsilon_A^s (v_{Az}^s - w_z) - \epsilon_B^s (v_{Bz}^s - w_z)] = 0 \tag{15}$$

for a one-dimensional system.

Between the slurry and the filter cake in the cake filtration process the fluid-phase mass discontinuity can be written as

$$(\epsilon^f (v_z^f - w_z))_{\substack{\text{SLURRY} \\ z=L_c}} = (\epsilon^f (v_z^f - w_z))_{\substack{\text{CAKE} \\ z=L_c}} \tag{16}$$

where the subscripts SLURRY and CAKE indicate which region side of the boundary that the term represents. The subscript $z = L_c$ indicates the z-position of the boundary.

As the slurry encounters the boundary between the cake and the slurry, solid particles are added to the cake surface, which causes the surface to move with time. This movement is represented by $(w_z)_{z=L_c}$.

At the cake–medium, medium–plate, and plate–filtrate region boundaries the boundary is normally stationary and (w_z) is zero. This also implies that the solid phase velocity is zero. For these boundaries the fluid phase mass discontinuity balance has the form

$$[\epsilon_A^f v_{Az}^f - \epsilon_B^f v_{Bz}^f] = 0 \tag{17}$$

If the filter medium is very thick and compressive, then it may be possible to measure the movement of this boundary. In this case the boundary velocity, (w_z), would be nonzero. Also, if the solid particles can penetrate into the medium or plate or bleed through into the filtrate for these latter three boundaries, then the solid phase velocities also would be nonzero.

At the plate–filtrate boundary, Eq. (17) translates into a relation between the fluid phase velocity, (v_z^f), and the volumetric flow rate, Q:

$$A(\epsilon^f v_z^f)_{\substack{\text{PLATE} \\ z=0}} = -Q \tag{18}$$

where the minus signs account for the fluid velocity in the minus z-direction. Furthermore, through Eqs. (7) and (17) for zero solids velocity in the plate and medium, Eq. (18) extends to the medium–plate and the cake–medium boundaries to obtain

$$A(\epsilon^f v_z^f)_{\substack{\text{MEDIUM} \\ z=L_p}} = A(\epsilon^f v_z^f)_{\substack{\text{CAKE} \\ z=L_m}} = -Q \tag{19}$$

Before leaving the mass discontinuity balances for cake filtration, there is a useful relationship for the rate of cake growth that makes use of the mass discontinuity conditions listed above. Starting with the filter cake region, the fluid phase mass balance in integral form is

$$\int_{L_m}^{L_c} \left(\frac{\partial \epsilon^f}{\partial t} + \frac{\partial(\epsilon^f v_z^f)}{\partial z} \right) dz = 0 \tag{20}$$

which can be integrated by applying the fundamental theorems of calculus and the Leibnitz formula (13) to obtain

$$\frac{d}{dt}\int_{L_m}^{L_c} \epsilon^f dz - (\epsilon^f w_z)_{\substack{\text{CAKE} \\ z=L_c}} + (\epsilon^f w_z)_{\substack{\text{CAKE} \\ z=L_m}} + (\epsilon^f v_z^f)_{\substack{\text{CAKE} \\ z=L_c}} - (\epsilon^f v_z^f)_{\substack{\text{CAKE} \\ z=L_m}} = 0 \tag{21}$$

The integral in the first term of Eq. (21) is equal to the cake height, ($L_c - L_m$), times the cake average porosity, ϵ^{f*}. The product rule of calculus can then be applied to separate the time derivative of the product $(L_c - L_m)\epsilon^{f*}$.

Also, the fluid phase discontinuity conditions in Eq. (16) at $z = L_c$ and Eq. (19) at $z = L_m$ are applied to Eq. (21) to relate the boundary terms to measurable quantities at the boundaries. Now Eq. (21) becomes

$$(L_c - L_m)\frac{d\epsilon^{f*}}{dt} + \epsilon^{f*}(w_z)_{z=L_c} + (\epsilon^f(v_z^f - w_z))_{\substack{\text{SLURRY} \\ z=L_c}} + Q/A = 0 \tag{22}$$

where the time derivative of L_c is equal to the boundary velocity, $(w_z)_{z=L_c}$.

As an approximation, the fluid and solid phase velocities are assumed to be the same in the slurry above the cake. Hence these two velocities must be equal to the superficial velocity or the flow rate divided by the cross-sectional area:

$$v_{z\ \text{SLURRY}}^f = v_{z\ \text{SLURRY}}^s = -Q/A \tag{23}$$

Furthermore, Willis et al. (14) and Willis and Tosun (15) report the time rate of change of the cake average porosity to be negligible for most cakes.

Applying the above simplifications to Eq. (22), the macroscopic fluid phase mass balance takes the form

$$(w_z)_{z=L_c}(\epsilon^{f^*} - \epsilon^{f}_{SLURRY}) + (1 - \epsilon^{f}_{SLURRY})Q/A = 0 \tag{24}$$

Defining the function G as the instantaneous rate of cake growth to the filtrate volumetric flow rate as

$$G(t) = \frac{A(w_z)_{z=L_c}}{Q} \tag{25}$$

then Eq. (24) becomes

$$G(t) = \frac{(1 - \epsilon^{f}_{SLURRY})}{(\epsilon^{f}_{SLURRY} - \epsilon^{f^*}_{CAKE})} \tag{26}$$

Hence, Eqs. (25) and (26) provide a way of estimating the rate of cake growth, $(w_z)_{z=L_c}$, as a function of the slurry and cake average porosity.

MOMENTUM DISCONTINUITY BALANCES FOR CAKE FILTRATION

The region momentum discontinuity balance is given by

$$[\rho_A \mathbf{v}_A(\mathbf{v}_A - \mathbf{w}) + P_A\boldsymbol{\delta} + \boldsymbol{\tau}^s_A - \rho_B \mathbf{v}_B(\mathbf{v}_B - \mathbf{w}) - P_B\boldsymbol{\delta} - \boldsymbol{\tau}^s_B]\cdot n_{AB} = 0 \tag{27}$$

For the one-dimensional process that is being considered here, and where the inertial terms are insignificant, Eq. (27) reduces to

$$[P_A + \tau^s_{Azz} - P_B - \tau^s_{Bzz}] = 0 \tag{28}$$

In the filtration process considered here, the same fluid is in the regions on each side of the boundaries. This results in the pressures and stresses on each side of the boundaries being the same, and the pressure and stresses can be decoupled in Eq. (28) to obtain

$$P_A = P_B \tag{29}$$

and

$$\tau^s_{Azz} = \tau^s_{Bzz} \tag{30}$$

At the slurry–cake boundary the solid phase structure in the slurry is fluidlike and cannot support any compressive stress. The stress on the solid phase at this boundary is therefore zero:

$$\tau^s_{zz\ \underset{z=L_c}{SLURRY}} = 0 \tag{31}$$

Also, at the plate–filtrate boundary the stress on the solid phase *in the z-direction* is similarly zero. Some stresses must occur within the plate to transmit the load of the stress at the medium–plate boundary to the walls of the filter assembly. The total force by the walls to hold the plate stationary, F_{PLATE}, is related to the solid stress at the medium–plate boundary by

$$F_{PLATE} = A(\tau^{S}_{ZZ})_{\substack{PLATE \\ z=L_p}} \tag{32}$$

where A is the cross-sectional area to flow for the filter assembly.

When the fluids on either side of the boundary are immiscible, as in the drainage process shown in Fig. 2, then the discontinuity balances are different than those described above. The drainage boundary condition is now considered.

DISCONTINUITY BALANCES FOR CAKE DRAINAGE

Many of the discontinuity balances derived above also apply to the drainage process. At the gas–cake boundary at $z = L_c$, the cake no longer grows due to particles from the slurry adding to the cake. Hence, this boundary may be stationary. However, this boundary may move with velocity $(w_z)_{z=L_c}$, as indicated in Eq. (16) where the gas phase replaces the slurry if there is significant swelling or shrinkage as the cake is drained.

The other difference is the additional discontinuity balance required for the drainage boundary shown in Fig. 2. Let the subscript GAS indicate the portion of the cake filled with the gas phase and let the subscript LIQUID indicate the liquid-saturated part of the cake. At this boundary, capillary forces cause the gas-phase pressure to differ from the liquid-phase pressure across this boundary. The pressure term, P, that appears in the continuum equations and the discontinuity equations refers to the measurable pressures of the continuous gas and liquid phases on each side of the boundary. It does not refer to the pressures within the discontinuous residual liquid droplets left behind in the cake because the free liquid is displaced by the gas phase.

The capillary forces at the drainage boundary cause a pressure difference between the gas and the liquid phases. This pressure difference is called the capillary pressure, P_{CAP},

$$P_{LIQUID} - P_{GAS} = P_{CAP} \tag{33}$$

From Eq. (28) the capillary pressure also relates the discontinuity in the stresses in the solid phase at the boundary. Combining Eqs. (28) and (33), the stresses are related by

$$[\tau^{S}_{GASzz} - \tau^{S}_{LIQUIDzz}] = P_{CAP} \tag{34}$$

The mass balances given in Eqs. (14) and (15) do not apply in general to the drainage boundary because there are now three phases to be accounted for: the solid phase, the liquid phase, and the gas phase. Equations (12) and (13) become

$$\rho = \epsilon^l \rho^l + \epsilon^g \rho^g + \epsilon^s \rho^s \tag{35}$$

$$\rho \mathbf{v} = \epsilon^l \rho^l \mathbf{v}^l + \epsilon^g \rho^g \mathbf{v}^g + \epsilon^s \rho^s \mathbf{v}^s \tag{36}$$

When Eqs. (35) and (36) are combined with the region mass balance in Eq. (11) for a one-dimensional process, we get

$$\begin{bmatrix} (\rho^l \epsilon^l v_z^l + \rho^g \epsilon^g v_z^g + \rho^s \epsilon^s v_z^s)_{\mathrm{GAS}} - (\rho^l \epsilon^l + \rho^g \epsilon^g + \rho^s \epsilon^s)_{\mathrm{GAS}} w_z \\ - (\rho^l \epsilon^l v_z^l + \rho^g \epsilon^g v_z^g + \rho^s \epsilon^s v_z^s)_{\mathrm{LIQUID}} + (\rho^l \epsilon^l + \rho^g \epsilon^g + \rho^s \epsilon^s)_{\mathrm{LIQUID}} w_z \end{bmatrix} = 0 \tag{37}$$

To make this equation more manageable, we note that the liquid phase that remains in region A after the gas has pushed out the free liquid, denoted by $(\rho^l \epsilon^l)_{\mathrm{GAS}}$, is the residual liquid that is trapped within the solid matrix and has the same velocity as the solid phase. Also, within the LIQUID region there is no gas phase present. Equation (37) simplifies to

$$\begin{bmatrix} (\rho^g \epsilon^g v_z^g + (\rho^l \epsilon^l + \rho^s \epsilon^s) v_z^s)_{\mathrm{GAS}} - (\rho^l \epsilon^l + \rho^g \epsilon^g + \rho^s \epsilon^s)_{\mathrm{GAS}} w_z \\ - (\rho^l \epsilon^l v_z^l + \rho^s \epsilon^s v_z^s)_{\mathrm{LIQUID}} + (\rho^l \epsilon^l + \rho^s \epsilon^s)_{\mathrm{LIQUID}} w_z \end{bmatrix} = 0 \tag{38}$$

Furthermore, in a drainage process the movement of the solid phase is usually insignificant compared to the movement of the fluid phases and the drainage boundary. Also, the volume fraction of the solid phase can be assumed to be the same on each side of the boundary. Hence, Eq. (38) simplifies to

$$[(\rho^g \epsilon^g v_z^g)_{\mathrm{GAS}} - (\rho^l \epsilon^l + \rho^g \epsilon^g)_{\mathrm{GAS}} w_z - (\rho^l \epsilon^l v_z^l)_{\mathrm{LIQUID}} + (\rho^l \epsilon^l)_{\mathrm{LIQUID}} w_z] = 0 \tag{39}$$

Finally, if the rate of mass transfer per unit area between the liquid phase and the gas phase (by evaporation) is significant at this boundary, then we decouple the phases in Eq. (39) by introducing the mass transfer term, E. The equations decouple as

$$\rho^g_{\mathrm{GAS}} \epsilon^g_{\mathrm{GAS}} (w_z - v^g_{\mathrm{GASz}}) = E \tag{40}$$

$$\rho^l_{\mathrm{LIQUID}} \epsilon^l_{\mathrm{LIQUID}} (w_z - v^l_{\mathrm{LIQUIDz}}) - \rho^l_{\mathrm{GAS}} \epsilon^l_{\mathrm{GAS}} w_z = E \tag{41}$$

Equation (40) relates the gas velocity to the drainage boundary velocity and the rate of mass transfer between the gas and liquid phases at the boundary. If no mass transfer occurs, then the gas velocity and the drainage boundary velocity are equal. Equation (41) relates the liquid velocity to the velocity of the drainage boundary velocity, the rate of mass transfer between the gas and liquid phases, and the amount of residual liquid left behind.

A pressure difference between the cake and medium regions such as the capillary pressure given in Eq. (33) can also occur during surface clogging of the medium. This phenomena may be related to cake compressibility and depth clogging of the medium, but these latter effects are modeled by appropriate constitutive relations in Eqs. (4) and (5). Surface clogging is a boundary effect, such as straining at the surface in which cake particles plug pores of the medium just at the boundary, and results in a pressure difference that must be accounted for with a pressure drop function analogous to P_{CAP} as applied in Eq. (33).

CONCLUSIONS

The work here shows how the general jump discontinuity balance is applied between multiphase regions to obtain discontinuity balances for the gas, liquid, and solid phases. Specific discontinuity balances are obtained for the processes of cake filtration and liquid drainage for a filter cake. The effects of mass transfer and capillary pressure are accounted for in the discontinuity balances for the drainage boundary which do not normally appear in the discontinuity balances for the multiphase regions in the filter cake. These discontinuity balances can now be applied to continuum models for solving the continuum equations for each separate multiphase region.

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NOTATION

A	cross-sectional area of filter assembly
A_{AB}	area of boundary between multiphase regions A and B
E	rate of mass transfer between the gas and liquid phases across the drainage boundary
F_z^{d}	drag force between phases

F_{PLATE}	force
G	ratio of cake growth rate to filtrate rate
$\mathbf{i}$	flux of arbitrary property ϕ
L_c	z-position at slurry–cake boundary
L_m	z-position at cake–medium boundary
L_p	z-position at medium–plate boundary
$\mathbf{n}$	area normal vectors in Fig. 3
P	piezometric pressure defined by Eq. (6)
P^f	fluid phase pore pressure
P_{CAP}	capillary pressure
Q	volumetric flow rate
t	time
V_A, V_B	A and B region volumes in Fig. 3
v_z^α	α-phase average velocity
w_z	velocity of boundary
z	axial position as measured from the plate–filtrate boundary
ϵ^α	α-phase volume fraction
ϵ^{α^*}	region average α-phase volume fraction
ρ^α	α-phase intrinsic density
τ_{zz}^s	stress on solid phase matrix
$\boldsymbol{\delta}$	Kronecker delta
ϕ	arbitrary material property

Superscripts/Subscripts

α, f, s	α-phase, fluid phase, solid phase quantity
g, l	gas and liquid phase quantities at the drainage boundary
z	z-component of a vector or tensor
CAKE	quantity evaluated in the cake region
MEDIUM	quantity evaluated in the medium region
PLATE	quantity evaluated in the plate region
SLURRY	quantity evaluated in the slurry region
GAS	quantity evaluated in the gas-occupied portion of the draining cake
LIQUID	quantity evaluated in the liquid-saturated portion of the draining cake
z=L	quantity evaluated at boundary at $z = L$

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